CYLINDRICAL WAVES IN A COLLISIONLESS PLASMA

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Experimental and theoretical studies are now being performed on finite-amplitude waves and shock waves propagating in collisionless plasmas. A theory of these plane stationary waves was constructed in [1]. Of great interest is a consideration of nonstationary wave motion in plasmas where the waves are excited by a magnetic piston, i.e., a magnetic field growing at the plasma-vacuum boundary according to some particular law.

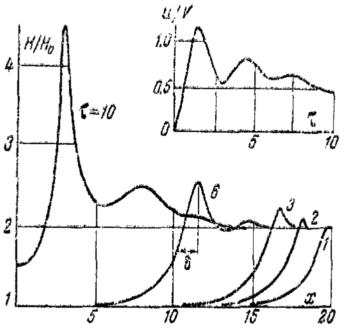

Fig. 1. Space profile of magnetic field at different moments in time and piston velocity as a function of time for "exponential" mode $R = 20c/\omega_{0e}$, $\nu = 0.5\omega_{+}$.

In this study we consider the principles governing the motion of strong cylindrical waves propagating in a cold collisionless plasma and compare the experimental results with the results obtained by numerically solving the appropriate system of equations on an electronic computer. In the experiments a plasma with frozen longitudinal magnetic field H_0 was rapidly compressed by an external field $H_{\sim}(t)$ created by a shock coil enclosing the plasma. The wave profile was studied from the readings of magnetic probes placed at different distances from the system axis. A more detailed description of the experimental technique and the results obtained is given in [2]. In the "machine experiment" the plasma is assumed to be quasi-neutral, completely ionized, and cold ($nT \ll H^2/8\pi$) and is described by the equations of macroscopic motion of the electron and ion components and by the Maxwell equations with self-consistent electromagnetic fields. If the wave propagates strictly across the magnetic field, the problem is one-dimensional (all quantities depend upon the radius r and the time t).

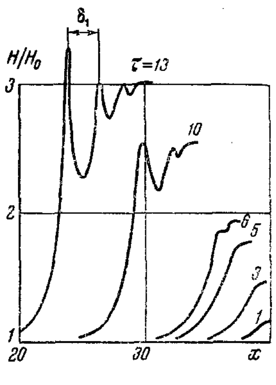

Fig. 2. Space profile of magnetic field at different moments in time for "sinusoidal" mode $R = 40c/\omega_{0e}$, $\nu = 0$.

The mathematical statement of this problem and some calculation results are given in [3]. To excite waves propagating across the magnetic field the surface of the plasma must be parallel to H_0 and the pressure $H_{\sim}^2/8\pi$ along this surface must be uniform; this is determined by the geometry of the conductors. For a cylindrical solenoid this nonuniformity depends upon the ratio of the radius R to the length l.

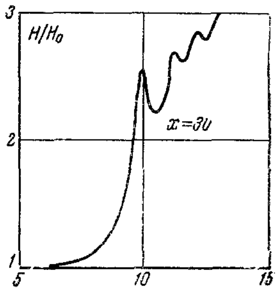

Fig. 3. Time profile of field at $x = r\omega_{0e}/c = 30$ ($R = 40c/\omega_{0e}$, $\nu = 0$).

The presence of a uniform column of conducting plasma considerably improves the uniformity of magnetic pressure. If the gap a separating the surface of the conductor and the plasma is fairly small in comparison with its length, the magnetic field pressure at the plasma boundary will be everywhere uniform except for distances on the order of a/l at the ends of the system, even if the ratio R/l is large. In this case the magnetic piston moves across the magnetic field. If, before compression, the plasma is significantly nonuniform, the wave will not propagate strictly across the magnetic field and the problem will not be one-dimensional (this has been shown experimentally). However, even in this case, we can use the one-dimensional model to perform calculations if, as in [4], we introduce into the equations some effective average angle θ between the direction of the field H_0 and the plane of the wave front. In this article we examine the transverse propagation of cylindrical waves.

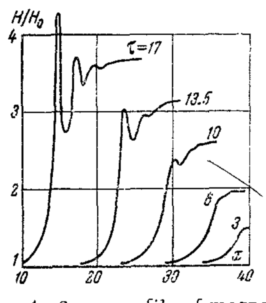

Fig. 4. Space profile of magnetic field. $R = 40c/\omega_{0e}$, $\nu = 0.5\omega_{+}$.

NOTATION

H_0 is the constant magnetic field; $H_{\sim}$ is the variable external magnetic field; ρ is the mass density of plasma; $T_{i(e)}$ is the ion (electron) temperature; U is the wave velocity; M is the Mach number; V is the Alfvén velocity; ν is the effective collision frequency; ω_{0e} is the plasma electron frequency; Δ is the wave front width; σ is the effective plasma conductivity; w is the diffusion rate; ω_{+} is the hybrid frequency; u is the piston velocity; δ is the oscillation width; R is the initial radius of plasma filament; Ω_0 is the plasma ion frequency.

1. Statement of the problem. If dissipation, i.e., the friction between plasma components, is not too great, the profile of the perturbation propagating through the plasma under the action of the magnetic piston becomes oscillatory: a wave train whose amplitude increases in the direction of motion will propagate in the plasma (as is shown in [1], this amplitude increase is possible up to $H = 3H_0$; the wave then "breaks" and motion becomes multistream). When there is no dissipation the number of oscillations continuously increases, and a stationary profile does not occur. If we allow for dissipation, a stationary shock

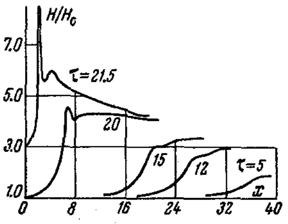

Fig. 5. Space profile of magnetic field: $R = 40\ c/\omega_{0e}$, $\nu = 5\omega_+$.

wave with an oscillatory structure is possible. It was very difficult to set up the laboratory conditions required for the generation of a stationary shock wave. For a cylindrical geometry it is not possible to realize the stationary process since wave motion toward the system axis leads to considerable changes in the wave (the amplitude increases sharply as the axis is approached). Here too, however, we can treat the wave as a "stationary" one when dissipation is present and the width Δ of the front formed is fairly small in comparison with the distance r to the axis. According to [1], the front width or the total damping length is given by

$$\Delta \approx U / \nu . \tag{1.1}$$

Here, U is the velocity of the shock wave relative to the unperturbed plasma, and ν is the effective collision frequency. Therefore, we can assume the wave profile is, to some degree, stationary when

$$U / \nu = MV / \nu \ll r , \tag{1.2}$$

where M is the Mach number. Naturally, the linear dimension of the oscillation must be small in comparison with the radius, i.e.,

$$c / \omega_{0e} \ll r \quad (\omega_{0e} = \sqrt{4\pi ne^2 / m_e}) .$$

If the system radius $r \approx 8$ cm, these requirements will be satisfied when the initial (unperturbed) plasma has the parameters

$$n_0 \sim (10^{12} - 10^{13})\mathrm{cm}^{-3}, \quad H_0 \sim (0.2-1) \quad \mathrm{kOe},$$
$$\nu \sim (10^8 - 10^9)\ \mathrm{sec}^{-1}, \quad M \lesssim 3 . \tag{1.3}$$

Moreover, we can assume the wave profile is stationary if the change in this profile (caused by the change in the magnetic field at the plasma boundary as the wave passes the probe) is negligible:

$$\frac{dH_\sim}{dt} \frac{\Delta}{U} \ll H_{\max} . \tag{1.4}$$

$H_\sim(t)$ is the external pulsed field and $H_{\max}$ is the field amplitude in the wave (for sinusoidal changes in $H_\sim$). This condition is satisfied, to a reasonable degree, when

$$H_\sim \sim (2-8)\ H_0, \qquad \omega \sim (2-5) \cdot 10^6 \mathrm{sec}^{-1} . \tag{1.5}$$

Calculations were also performed for the parameters in (1.3) and (1.5). We now consider in detail the nature of the cylindrical waves and their generation.

2. Transient processes. By this term we mean effects occurring from the moment the magnetic field starts to built up (piston effect) to the point when the wave profile becomes stationary in the above sense. The specific nature of the transient processes, aside from nonlinear and dispersion effects, is determined by the following factors: 1) the law governing the increase in the external field $H_\sim = Af(t)$, where A is the amplitude; 2) the effective collision frequency ν (the rate of diffusion of the magnetic field depends upon this quantity); 3) the ratio of the system radius R to the characteristic dimension c/ω_{0e}.

When the piston effect begins, the boundary of the plasma column moves toward the axis and forms a cylindrical compression region. If magnetic field diffusion is ignored ($\nu = 0$) and the piston velocity u is greater than the Alfvén velocity $V = H_0/(4\pi\rho_0)^{1/2}$, the perturbation will grow without separating from the piston until the perturbation velocity exceeds the piston velocity. If the piston velocity u is less than the Alfvén velocity, small perturbations will continuously separate from the piston

The piston velocity u increases with time; therefore the perturbation profile droops more or less exponentially in the direction of the axis. If we allow for friction ($\nu \neq 0$), then magnetic field diffusion will take place; this is equivalent to additional transport of the "foot" of the perturbation with the diffusion velocity

$$w \sim \frac{c}{\sqrt{4\pi\sigma t}} \qquad \left(\sigma = \frac{ne^2}{m_e \nu}\right) .$$

As the perturbation continues to progress, the nonlinear effects leading to steepening of the perturbation profile become more pronounced. The initially exponential profile becomes bell-shaped. The first oscillation will detach itself from the piston when the local crest velocity is greater than V + w. Note that until this profile is established the "foot" velocity remains more or less constant and equal to V + w.

Figures 1–10 give the calculation results, Fig. 1 applying to the "exponential" piston $H_\sim(t) = H_0(1 + A(1 - e^{-\omega t}))$, and Figs. 2–10 to the "sinusoidal" piston $H_\sim(t) = H_0(1 + A \sin \omega t)$. For the "exponential" piston the velocity u of the plasma boundary rapidly increases, reaching the values

$$V = \frac{H_0}{\sqrt{4\pi n_0 m_i}} \qquad \text{when } t = t_0 \sim \frac{c}{\omega_{0e} V} .$$

At $t < t_0$, when $u < V$, a leading "foot" is formed. At $t = t_1 \gtrsim 2c/\omega_{0e}V$, when the wave velocity becomes greater than the piston velocity, the perturbation detaches from the piston. In this mode, the perturbation for $t_0 < t < t_1$ has dimension roughly equal to c/ω_{0e}; therefore, at $t = t_1$ the oscillation formed will become detached. The transient processes occur in a time $0 < t \lesssim 2c/\omega_{0e}V_A$. The "sinusoidal" piston, which approaches the actual conditions of the experiment more closely, was studied in much greater detail: for different external field amplitudes ($A = 4, 8$), different relative initial plasma-column radii ($R\omega_{0e} = 20, 30, 40$) (which essentially correspond to different initial densities), and for different values of the dissipation ($\nu/\omega_+ = 0, 0.5, 1, 2, 3, 5$).

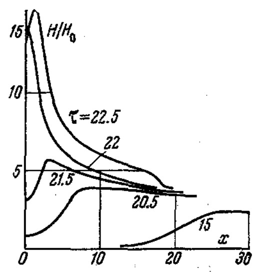

Fig. 6. Space profile of magnetic field: $R = 40\ c/\omega_{0e}$, $\nu = 5\omega_+$.

For the sinusoidal piston the external field does not grow as rapidly, which leads to a greater "pulling out" of the "foot". Figures 2, 4, 5, and 6 illustrate the above sequence of processes leading to the establishment of equilibrium steepness of the profile. We should note that the first oscillation will detach after the piston has traveled a certain distance depending upon the dissipation and the rate of increase in the external field.

Experimentally it is convenient to examine this phase for opposed fields H and $H_\sim$ since in this case the region of the wave and the piston are always clearly defined and the features associated with field orientation do not qualitatively affect the nature of the process. Figure 11 shows oscillograms obtained from probes placed at different distances from the chamber wall. Probe 1, placed at a distance ~1 cm, registers the sharp field discontinuity region although the compression wave has still not formed. Probe 2, placed at a distance approximately equal to one-half the chamber radius, registers the oscillation formed, after whose separation, $U > V$, $u > V$ ($V \approx 10^7$ cm/sec, $u \approx 1.5 \cdot 10^7$ cm/sec, $U \sim 4 \cdot 10^7$ cm/sec).

The transient process depends considerably upon the amount of dissipation. If $M\omega_+/\nu \lesssim 1$, the profile is aperiodic and its width is several times the dispersion length c/ω_{0e}. This condition is satisfied for low dissipation ($\nu/\omega_+ = 5$, Figs. 6 and 7). If $M\omega_+/\nu > 1$, which occurs when $\nu/\omega_+ \lesssim 3$, the profile will possess an oscillatory structure. The wave front becomes stationary when the number of oscillations generated is equal to $M\omega_+/\nu$.

Several oscillations may form, however, when the magnetic pressure at the boundary continues to increase after the first perturbation detaches; otherwise a single wave will propagate through the plasma.

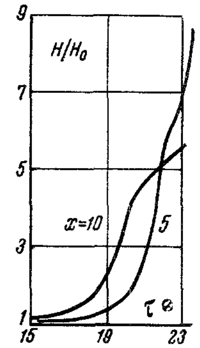

Fig. 7. Time profile of magnetic field at $x = 5$ and $x = 10$: $R = 40\ c/\omega_{0e}$, $\nu = 5\omega_+$.

To establish the profile it is also necessary that the time elapsed from the moment the first oscillation is formed t_1 to the moment of wave "breaking" t^* or pile-up t_* be sufficient for the formation and detachment of all the oscillations that which must appear in the stationary profile, i.e.,

$$t^*\ (t_*) - t_1 > \Delta / U \sim 1/\nu. \qquad (2.1)$$

The times t^* and t_1 are functions of the external field amplitude A and the dissipation ν/ω_+:

ν/ω_+	$t_1 \frac{\omega_{0e}V}{c}$	$t^* \frac{\omega_{0e}V}{c}$	H_*/H_0	M_*	$\frac{\delta\omega_{0e}}{c}$
0	—	—	3.3	2.15	0.6
0.5	6	17	4.2	2.6	0.5
1	8	19	4.5	2.75	0.6
2	13	21.5	—	—	—
5	—	22.5	—	—	—

These data are obtained from the solution for the case $R = 40\ c/\omega_{0e}$, $A = 8$.

Consider the problems associated with "breaking" of the wave and its reflection from the system axis. Calculations performed on the basis of the model discussed in the introduction show that in some particular time interval $t = t_*$, depending upon the system parameters, the wave amplitude reaches a value at which the plasma density ρ strongly increases ($1/\rho \to 0$). This time $t = t_*$ can clearly be regarded as the moment preceding the moment of wave "breaking." Depending on the system parameters (field amplitude, initial radius and dissipation) two cases are possible: 1) at $t = t_*$ the entire wave (including the foot) is remote from the axis (as yet no pile-up); 2) wave "breaking" takes place almost at the same time as the sharp increase in field at the axis as a result of pile-up. In the first case the wave parameters before "breaking" (M_*, H_*, δ) are close to the values obtained from the theory of stationary plane waves (for a stationary plane wave $H_* = 3H_0$, $M_* = 2$).

In the second case cylindricity becomes important and the "foot" strongly increases. Here the wave parameters before "breaking" are

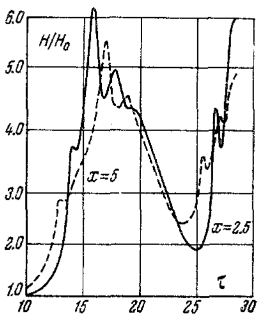

Fig. 8. Time profile of magnetic field at $x = 2.5$ and $x = 5$: $R = 20c/\omega_{0e}$, $\nu = 0.5\omega_+$.

quite different from the theoretical values for plane waves which, naturally, cannot be used in this case. This situation occurs for high dissipation ($\nu/\omega_+ \gtrsim 2$). For example, when $\nu/\omega_+ = 5$ (Fig. 6) the formation of an equilibrium profile slope is completed almost at the same time as wave pile-up because of the strong diffusion of the magnetic field and "breaking" is registered after the wave is reflected from the system axis.

If condition (2.1) is not satisfied (which occurs when dissipation is lower and the radius of the system has its initial value), "breaking" of the leading oscillation continues until a loop is formed; in this case we cannot speak of a stationary profile even in a limited sense.

If the parameters of the system are such that the wave can approach the axis without "breaking," pile-up will occur and the magnetic field near the axis strongly increases (Fig. 6). After pile-up the wave is reflected: at the system axis the field attenuates and at the periphery a divergent wave is registered; this wave again has a tendency to become steeper in the direction of propagation. As follows from calculations, at high dissipation the wave can "break" even after pile-up and reflection from the axis. For example, for $A = 8$, $R = 40\ c/\omega_{0e}$, $\nu/\omega_+ = 5$, pile-up takes place at a time $t \lesssim 22\ c/\omega_{0e}V$ and "breaking" takes place at $t \gtrsim 22.5\ c/\omega_{0e}V$. Figure 8 shows the time profile of the magnetic field for $R = 20\ c/\omega_{0e}$, $\nu/\omega_+ = 0.5$. In this case wave reflection from the axis after pile-up and the repeat compression of the column with the excitation of a new cylindrical wave are traced.

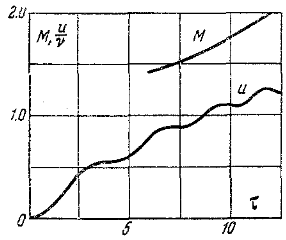

Fig. 9. Piston and wave velocities as functions of time for no dissipation ($\nu = 0$).

3. Dynamics of "piston" and wave. Consider the motion of the plasma boundary under the effect of an external growing field in the "sinusoidal" case. We see that the nature of motion of the boundary is closely connected with the formation of oscillations. This is quite noticeable when there is no dissipation. Figure 9 shows the boundary velocity as a function of time when $R = 40\ c/\omega_{0e}$, $A = 8$, $\omega = 0.02\omega_+$, $\nu = 0$. This relationship is clearly defined and is not monotonic. The boundary first begins to accelerate since the external magnetic field increases. Acceleration then ceases and the velocity remains more or less constant over some period of time. Then the velocity again increases; thus, the curve $u = u(t)$ has several inflection points. The number of such velocity "cycles" corresponds to the number of oscillations formed.

The fact that the relationship u(t) is not monotonic is associated with the increase in field strength near the boundary when the wave velocity U and the piston velocity u are commensurate. The first inflection point corresponds to the formation of the first oscillation on the profile ($t \approx 4.5\ c/\omega_{0e}V$). Here the magnetic field near the inside of the boundary is equivalent to the external field. When the first oscillation detaches, the magnetic field in the plasma near the boundary decreases and the piston again accelerates ($t \approx 5.5\ c/\omega_{0e}V$) under the effect of magnetic pressure. This leads to the formation of a second oscillation, etc. When there is dissipation, the dependence of the piston velocity upon time becomes more monotonic and when $\nu \gtrsim \omega_+$ these piston "vibrations" disappear (Fig. 10).

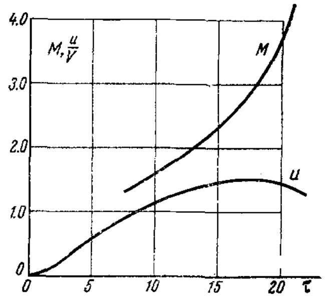

Fig. 10. Piston and wave velocities as functions of time for high dissipation ($\nu = 5\omega_+$).

The general form of u(t) in the presence of dissipation is almost independent of its magnitude. The velocity of the boundary increases monotonically reaching a maximum $u_{max} \approx 1.5V$ at $t \approx (18\text{–}19)\ c/\omega_{0e}V$, and then begins to decrease. This is due to the fact that as the piston approaches the axis the magnetic field in the plasma increases and at some moment the piston begins to slow down. This is particularly noticeable when there is a great deal of dissipation (Fig. 10) and wave pile-up and reflection take place before "breaking."

On the basis of the change in boundary velocity with time we can draw some conclusions as to manner in which the transient processes depend upon dissipation. When there is no dissipation the wave forms immediately at the plasma boundary and each oscillation detaches under the effect of the piston near the boundary. The oscillations in magnetic field amplitude must naturally lead to a change in piston velocity. If dissipation is quite high, the perturbation becomes broader and flatter as a result of diffusion; nonlinear effects begin to act later than in the absence of dissipation when the perturbation moves away from the piston. As a result of this, separation of the oscillation takes place at a relatively great distance from the plasma boundary and does not affect the magnetic field in the plasma near the piston (and therefore the piston velocity). Therefore the relationship u(t) is monotonic when $\nu/\omega_+ \gtrsim 1$. We also note that the maximum piston velocity depends upon the plasma density and the external field amplitude.

Figures 9 and 10 also represent the mean velocity of the cylindrical wave as a function of time; this relationship is calculated from the displacement of the point on the leading wave front where the magnetic field is one-half its maximum value.

4. Structure of Cylindrical Waves. We shall consider the structure of cylindrical waves propagating toward the system axis on the basis of the obtained numerical solutions (Figs. 2–8) and experimental results. In this respect the following problems are of interest: under what conditions will the wave have an oscillatory structure; how many oscillations will there be in the stationary wave; and what is the width of these oscillations? The presence of an oscillatory structure depends upon the amount of dissipation: when there is little dissipation ($\nu/\omega_+ \gtrsim 3$ for $A = 8$, $R = 40\ c/\omega_{0e}$) the cylindrical wave consists of a series of oscillations; at high dissipation ($\nu/\omega_+ > 3$) oscillations do not appear and the wave profile is primarily determined by the cylindrical geometry (the magnetic field and density increase as the wave approaches the axis). The oscillatory or monotonic nature of profile also depends upon the amplitude of the external magnetic field. At low dissipation, the oscillatory structure will disappear when the field amplitude is low. For example, when $A = 4$, the profile is not oscillatory for $\nu/\omega_+ = 2$ whereas an oscillatory structure does occur when $A = 8$ and the dissipation is the same. This situation can be explained by the fact that at low field amplitudes the piston velocity is lower than at high amplitudes and broadening of the profile owing to magnetic field diffusion cannot be compensated by the nonlinear steepening; only when this compensation occurs is it possible for a profile with an oscillatory structure to form.

When there is dissipation the front width of the plane shock wave with oscillatory structure is given by formula (1.1), i.e.,

$$\Delta \approx \frac{M\omega_+}{\nu} \frac{c}{\omega_{0e}}.$$

A comparison of the loop length Δ (which may be called the front width) as obtained from calculations with the value calculated from formula (1.1) shows that when we can assume that the wave is stationary ($\Delta \ll R$) these quantities are in fairly good agreement:

ν/ω_+	M	$M\omega_+/\nu$	Δ
0.5	2.5	5	5–6
1	2.8	2.8	3–4
2	3.9	2	2–3

These data were obtained by solving for the case $A = 8$, $R = 40\ c/\omega_{0e}$. When the amplitude A and initial radius are small, this agreement is not obtained, because the stationary process is not reached before wave pile-up. If the width of the leading front is determined by magnetic field diffusion (in the presence of dissipation), its order of magnitude must be

$$\delta \sim \frac{c^2}{4\pi\sigma U} = \frac{\nu}{\omega_+ M} \frac{c}{\omega_{0e}}. \tag{4.1}$$

It follows from calculations that when $\nu \lesssim 2\omega_+$ this quantity is much less than the dispersion length; therefore, in the cases considered (none of which has an oscillatory structure) the width of the leading front—more exactly, the width of the leading oscillation—is determined by dispersion effects. For fairly high dissipation ($\nu/\omega_i > 3$) the front width is best determined from formula (4.1). Note that when $\nu/\omega_+ = 5$, $A = 8$ the value $\delta \approx 2.7\ c/\omega_{0e}$ obtained from calculation is greater than the value computed from formula (4.1). This is due to the fact that the wave is not stationary before pile-up.

For no dissipation ($\nu = 0$) it is not possible to speak of the wave as being stationary, even in a restricted sense. In this case the profile consists of a series of successive solitary waves which continuously detach from the piston; therefore, the "loop" length continuously increases. The leading oscillations propagate with a greater velocity than those following, since their amplitude is greater and the distance δ_1 between successive oscillations increases in time:

$t\frac{\omega_{0e}V}{c}$	δ_1	δ_1'
7	2	—
8.5	2.7	—
10	2.7	1.7
11.5	3	2.1

These data are obtained by solving for the case $\nu = 0$, $R = 20\ c/\omega_{0e}$, $A = 8$. The second column shows the distance between the first and second oscillations while the third column shows the distance between the second and third.

When $\nu \neq 0$ a shock wave exists which can be stationary under certain conditions (when $\Delta \ll R$). This shock wave can differ from the train of solitary waves formed for $\nu = 0$ in the behavior of the loop length Δ and the distance between successive oscillations in time. As shown above, when there is no dissipation the quantities Δ and δ_1 increase continuously. When there is dissipation, however, $\delta_1 \approx$ const, $\Delta \approx$ const for the shock wave and outside the oscillatory loop, which has a limited dimension, the roughly constant level of the magnetic field is "drawn out" up to the piston. This roughly constant field region increases with time and the oscillatory region moves on ahead, while $\Delta \approx$ const.

The time profile of the magnetic perturbations, registered with a fixed probe, depends considerably upon the experimental parameters (wave velocity U, effective collision frequency ν, rate of growth of magnetic field $dH_\sim/dt$) and upon the position r of the probe. If

$$\Delta \sim \frac{U}{\nu} \ll r, \qquad \frac{\Delta}{U} \ll \frac{H_{\max}}{dH_\sim / dt}.$$

the form of the signal from the probe becomes similar to the space structure of the wave front (Fig. 3). The closer the probe approaches the system axis, the more the signal differs from the space structure of the front because of the rapid cumulative increase of the field in the front (Fig. 7). When the probe is too far away from the axis, only the early phase of wave build-up is registered, i.e., the apparent "aperiodic" profile or nonstationary loop. With change in the parameters (ν, initial density n_0, $H_\sim/H_0$) the optimum position of the probe changes. When the experimental parameters change, the signal from the probe changes more markedly than the space structure of the wave. It is necessary to note that the effective collision frequency ν in the actual experiment changes along the wave front and varies with time; this is not allowed for in the "machine" experiment. Moreover, in view of the great difficulty, the phase of the process after wave "breaking" has not been calculated. This imposes certain restrictions on the comparison of the experimental and theoretical results.

Figure 12 shows an oscillogram of the magnetic perturbations in a helium plasma obtained from a probe placed at a distance R/4 from the system axis. The experimental oscillograph had a passband of 200 mHz and could record fronts with a minimum duration

$$\sim \frac{c}{\omega_{0e} V M} = \frac{1}{\omega_r M},$$

which, under typical conditions, corresponds to several $\sec^{-9}$. For the time interval corresponding to the traveling wave (~R/U) the form of

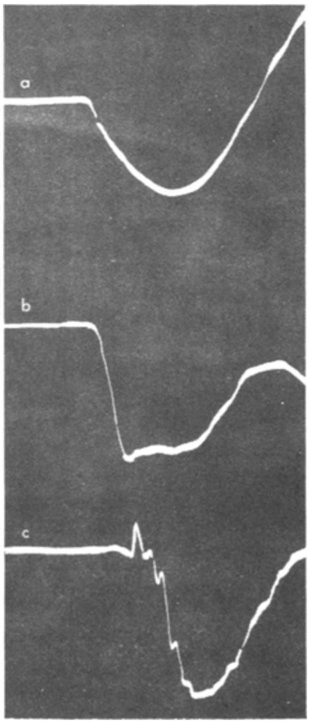

Fig. 11. Formation of initial perturbation at plasma periphery for opposed fields H_0 and $H_\sim$. The plasma is composed of helium with $n_0 = 3 \cdot 10^{13}\ \mathrm{cm}^{-3}$, $H_0 = 0.5$ kOe, $H_\sim = 4$ kOe: a) signal at magnetic probe in absence of plasma ($H_\sim(t))T/2$ = = 1.1 μsec); b) current sheet (piston) near chamber wall (probe 1); c) oscillation detached from piston (probe 2, $r \sim R/2$).

the observed probe signals is similar to the time picture obtained from calculation. When the shock coil current, exciting the "sinusoidal" piston, is "turned on," the magnetic field at the probe remains stationary for a certain time; then the probe registers a perturbation which rapidly increases to an amplitude comparable with max $H_\sim$. The signal shape shown in Fig. 12 can be interpreted as a sequence of crests on an increasing average level. The minimum width (time) of each crest approaches the value $c/\omega_{0e} U$. This oscillogram applies to minimum plasma densities ($n_0 < 10^{12}\ \mathrm{cm}^{-3}$) for which shock waves could be excited.

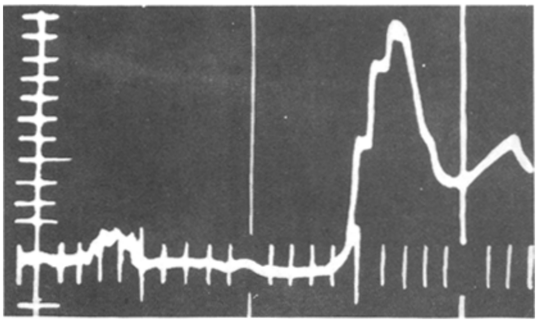

Fig. 12. Shock wave with minimum width leading front (the fields H_0 and $H_\sim$ have the same direction). Helium plasma with $n_0 \leqslant 10^{12}\ \mathrm{cm}^{-3}$, $H_0 = 280$ Oe, $H_\sim = 1600$ Oe. The fine scale time marks are 10^{-8} sec. After the first maximum (wave pile-up) we observe oscillations in the plasma column.

The width of the leading front, as follows from the above considerations, can be greater than the dispersion length c/ω_{0e} when $\nu > \omega$. However, if we allow for only pair-particle Coulomb collisions we obtain a value of $\nu \ll \omega_+$ (for $H_0 \sim 10^3$ Oe) for a highly ionized plasma with a density $n \sim 10^{12}-10^{13}\ \mathrm{cm}^{-3}$ and an initial temperature of ~10 eV. Therefore a broadening of the front is possible in the presence of turbulent dissipation due to the buildup of plasma oscillations resulting from an electron current across the direction of wave propagation. R. Z. Sagdeev considered a similar dissipation mechanism associated with the excitation of ion-acoustic oscillations in the wave front and leading to a considerable broadening of the front ($\delta \gg c/\omega_{0e}$). The plasma nonisothermicity ($T_e \gg T_i$), required for ion-acoustic oscillations to exist may result from electron heating through beam instability. The buildup conditions require that the electron drift velocity exceed the thermal velocity,

$$v > v_{T_e} = \sqrt{T_e / m_e},$$

and that the "existence time" of the drift current δ/U be sufficient for a considerable increase in the oscillations, i.e.,

$$\frac{\delta}{U} \sim \frac{c}{\omega_{0e} V M} \gg \frac{1}{\gamma} \sim \frac{1}{\Omega_0},$$

where γ is the small-perturbation growth increment. In the mode to which the oscillogram in Fig. 12 applies, the condition $\delta/U > 1/\Omega_0$ is not satisfied: this apparently explains the minimum front width observed in this case.

The condition $\delta/U \gg 1/\Omega_0$ may be satisfied when the wave velocity is reduced (for example, through an increase in plasma density); therefore, a front width roughly one order greater than the dispersion length is experimentally observed (see [2]).

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